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Implementation of Deep Scaffolding Based on Realistic Mathematics Education to Improve Thinking Errors of Students with Mild Intellectual Disabilities in Solving Numeracy Problems

Kadek Adi Wibawa*, I Made Dharma Atmaja
Universitas Mahasaraswati Denpasar
Bali, Indonesia

I Gde Dhika Widarnandana
Universitas Dhyana Pura
Bali, Indonesia

Abstract. Students with mild intellectual disabilities (MID) often struggle with numeracy concepts, particularly in representing geometric forms, performing arithmetic operations and applying mathematics to real-life contexts. This study aimed to examine how deep scaffolding grounded in Realistic Mathematics Education (RME) could remediate thinking errors of MID students in solving contextual numeracy problems. Using an interpretative qualitative approach with an exploratory single-case study design, one student (S1) with MID was selected from 61 participants identified through the Culture Fair Intelligence Test (CFIT). Data were collected through contextual numeracy worksheets, semi-structured interviews and observation, then analysed thematically using the mindful–meaningful–joyful scaffolding framework. The results revealed a significant transformation in S1's cognitive and emotional engagement. Initially, S1 was unable to represent or solve contextual problems. Through mindful scaffolding, S1 identified and corrected reasoning errors; meaningful scaffolding helped link prior experiences to mathematical contexts; and joyful scaffolding fostered confidence and reflective awareness. By the final session, S1 successfully solved multi-step numeracy tasks and demonstrated improved metacognitive and spatial reasoning abilities. This study highlights that deep scaffolding, when integrated with RME, not only enhances conceptual understanding but also promotes emotional resilience and reflective learning in students with MID. Theoretically, it expands Freudenthal's progressive mathematisation by incorporating affective and metacognitive

*Corresponding author: Kadek Adi Wibawa; *adiwibawa@unmas.ac.id*

dimensions. Practically, it offers an inclusive pedagogical model that can guide teachers in designing empathetic, contextual and transformative numeracy instruction for diverse learners.

Keywords: deep scaffolding; realistic mathematics education; numeracy; inclusion; mild intellectual disabilities

1. Introduction

Numeracy, the ability to understand and use numbers, is a fundamental life skill that determines individuals' capacity to make informed decisions (Sobkow, Olszewska, & Traczyk, 2020; Sobkow et al., 2025), solve problems effectively (Xiao et al., 2019; Yustitia, Kusmaharti, & Wardani, 2025), and participate in social and economic activities (Geiger & Schmid, 2024; Zehner et al., 2024). However, for students with mild intellectual disabilities (MID), developing numeracy competence remains an enduring challenge.

Studies have shown that these students experience significant delays in numerical magnitude representation (Brankaer, Ghesquière, & de Smedt, 2013) and struggle to connect mathematical concepts to real-life contexts (Cheong, Walker, & Rosenblatt, 2017; Bouck et al., 2018). In the context of inclusive education in Indonesia, such limitations are often compounded by limited teacher preparation, lack of adaptive learning models, and insufficient documentation of effective instructional interventions for learners with MID. Consequently, inclusive numeracy learning in Indonesian classrooms remains largely procedural, focusing on outcomes rather than the processes that enable conceptual understanding.

One instructional approach that has demonstrated effectiveness in improving mathematical understanding across diverse learners is Realistic Mathematics Education (RME). Rooted in Freudenthal's philosophy, RME positions real-life contexts as the foundation for mathematical learning, allowing students to construct meaning through problems they can visualise and relate to (Yilmaz, 2019; Das, 2020). Empirical evidence also indicates that RME can be adapted successfully for students with learning difficulties and special educational needs when teachers provide concrete representations and contextual mediation.

For instance, Chua (2021) found that RME improved conceptual understanding and positive dispositions toward mathematics among students with moderate learning challenges, while Huu et al. (2022) and Nurmasari, Nurkamto, & Ramli (2023) demonstrated its potential to enhance mathematical literacy and engagement through contextualised learning. Moreover, Listiawati et al. (2023) empirically confirmed that applying RME principles increased slow learners' competence in mathematics when instruction was modified using tangible and visual models, validating RME's flexibility in inclusive settings. These studies indicate that, although RME traditionally requires a cognitive transformation from situational understanding → model of → model on → formal mathematics (Van den Heuvel-Panhuizen & Drijvers, 2020), such progression can still occur in learners with mild intellectual disabilities when supported through concrete modelling and gradual scaffolding.

To make RME more accessible for students with mild intellectual disabilities (MID), additional structured support is required in the form of deep scaffolding—a gradual, reflective, and emotionally responsive intervention (Clements & Sarma, 2018 Sellars, 2017;). Deep scaffolding allows educators to identify learners' prior knowledge, diagnose specific cognitive barriers and design interventions that are both mindful and meaningful. Unlike surface-level scaffolding, which focuses mainly on procedural assistance, deep scaffolding fosters metacognitive awareness and emotional engagement, elements often neglected in traditional special education interventions. Within the inclusive education framework of Indonesia, such an approach holds significant promise for transforming both teacher practices and student learning outcomes.

Previous international studies have reported that scaffolding enhances conceptual development and problem-solving among students with learning difficulties (Reinhold et al., 2020; Malik et al., 2025). However, most existing studies, such as those by Reinhold et al. (2020), Long, Bouck, & Domka (2020), and Malik et al. (2025), tend to focus on procedural or technology-based scaffolding, emphasising performance improvement through digital tools or step-by-step guidance rather than exploring the reflective and personalised support processes essential for students with MID. Consequently, there remains limited empirical documentation of how deep scaffolding grounded in RME can be implemented to support numeracy learning among students with mild intellectual disabilities, especially within the inclusive educational context of Indonesia.

Therefore, this research is both practically urgent and theoretically significant. Practically, it responds to the growing inclusion of students with MID in Indonesian schools by providing teachers with a replicable model for numeracy instruction that is both inclusive and transformational. Theoretically, it contributes to the understanding of how RME-based deep scaffolding can bridge the gap between students' concrete experiences and abstract mathematical reasoning. The findings are expected to inform inclusive education policy, guide teacher training in adaptive pedagogy, and expand the global discourse on numeracy learning for students with cognitive disabilities.

2. Method

2.1 Research Type and Design of Research

This study employed an interpretative qualitative approach with an exploratory case study design. The approach was chosen to gain a deep understanding of the meaning, experience, and interaction process between the researcher (teacher) and a student with Mild Intellectual Disability (MID) during numeracy learning based on Realistic Mathematics Education (RME) and the Deep Scaffolding model. An exploratory case study design was deemed suitable for capturing the dynamics of individualised learning, scaffolding interactions and reflective learning behaviour as they naturally occur in authentic classroom contexts. The researcher acted as a facilitator and participant observer, providing instructional support and documenting the student's responses during the learning process.

The research was conducted at a private elementary school in Bali Province, which was officially designated as a PISA 2025 sample school by Kementerian Pendidikan Dasar dan Menengah. The school was selected purposively because it represents inclusive educational practice in Indonesia and participates in the PISA (Programme for International Student Assessment) numeracy assessment, reflecting the country's real classroom conditions and learning outcomes in global assessment frameworks.

In the preliminary stage, 61 upper-grade students (Grades 9) participated in a psychological assessment using the Culture Fair Intelligence Test (CFIT) administered by a licensed educational psychologist. The results identified eight students (13.1%) with IQ scores between 50 and 69, classified as Mild Intellectual Disability (MID) according to the Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (American Psychiatric Association, 2013). Among the eight students, there were five males and three females.

From these eight students, one participant (coded as S1) was selected as the primary case for an in-depth analysis using a single-case study design based on the following criteria: demonstrated sufficient verbal communication skills to engage in interviews and learning sessions, maintained consistent attendance throughout the intervention period, and exhibited the most representative profile of numeracy difficulties among the MID participants.

2.2 Research Instruments

Two main instruments were utilised: a contextual numeracy worksheet and a semi-structured interview protocol.

2.2.1 Contextual Numeracy Worksheet

The worksheet was designed following Realistic Mathematics Education (RME) principles, situating mathematical problems in real-life contexts familiar to students—such as counting money, comparing quantities and measuring length. The tasks were structured according to three levels of complexity aligned with Programme for International Student Assessment (PISA) numeracy indicators (OECD, 2022): identifying relevant information and understanding contextual problems, formulating mathematical representations and strategies, and interpreting and evaluating mathematical results within real-world contexts.

The instrument development involved three systematic phases: Content validation by two mathematics education experts and one educational psychologist, Limited pilot testing with two students identified as having mild learning difficulties to assess clarity and contextual appropriateness, and reliability testing through inter-rater agreement on students' numeracy comprehension scores.

The validation results showed a content validity index of 92% (very high category), and the reliability coefficient from the pilot test was 0.86 (high reliability), indicating strong internal consistency and suitability for assessing contextual numeracy among MID students.

The following are the numeration questions used by researchers:

POWDERED MILK AT MAXIMUM PRICE

A business owner has several products, one of which will be shipped to customers. The product is powdered milk in cans (sealed tubes) packaged in cardboard boxes measuring 60 cm x 30 cm x 45 cm. The product is placed upright inside the boxes to maintain stability during packaging and shipping.

There are three types of powdered milk cans that will be shipped:

- Small powdered milk cans with a diameter of 10 cm and a height of 12 cm
- Medium powdered milk cans with a diameter of 13 cm and a height of 18 cm
- Large powdered milk cans with a diameter of 14 cm and a height of 20 cm

If the price of the powdered milk is in accordance with the following table:

Type of powdered milk	Price	Berat
Small powdered milk	Rp 60.000,00	400 gram
Medium powdered milk	Rp 180.000,00	1,5 kg
Large, powdered milk	Rp 230.000,00	1,8 kg

What is the maximum amount of powdered milk that can be packed into a carton to achieve the maximum price? Explain your answer, including the types and quantities of powdered milk that will be packed into the carton, the total price and the rationale for this maximum price.

2.2.2 Semi-structured Interview Protocol

The interview protocol was developed based on the Deep Scaffolding framework, which comprises three interconnected domains: mindful scaffolding, helping students recognise their misconceptions and cognitive barriers, meaningful scaffolding, connecting prior knowledge to new contexts and strengthening conceptual understanding, and joyful scaffolding, fostering positive emotions, motivation and reflective engagement in learning.

The interviews were conducted individually (one-on-one) between the researcher (coded as R) and the student (S1) for 40–60 minutes per session. Each question was designed to elicit cognitive reasoning, self-reflection and emotional responses during the scaffolding process.

Examples of questions include:
1. Mindful Scaffolding (MiScaff): “How did you solve this problem? What was the first thing you thought about when reading it?”
2. Meaningful Scaffolding (MeScaff): “Why did you choose that strategy? Can you think of another way to solve it?”
3. Joyful Scaffolding (JoyScaff): “How did you feel when you found the answer?”

The interview guide was validated by two special education experts and one educational psychologist. The inter-rater reliability coefficient was 0.88 (high), confirming consistency in interpreting the qualitative data collected through the interviews.

2.3 Data Collection
The data collection took place over three weeks, consisting of four systematic stages:

- Participant Identification and Selection: The CFIT assessment was administered to all 61 students. Based on the results, eight students were classified as MID (IQ 50–69), and one student (S1) was purposively selected for the single-case study.
- Implementation of One-on-One Intervention and Observation: All interactions were audio-video recorded and supplemented with field observation notes to capture verbal expressions, gestures and problem-solving strategies in detail.
- Interviews and data triangulation: semi-structured interviews were conducted between R and S1 (40–60 minutes) to explore cognitive and emotional reflections. The result of interview are transcribed verbatim and verified through member checking to confirm accuracy of interpretation.
- Data Management and Confidentiality: All data were stored digitally using an anonymised coding system:
  R = Researcher/teacher
  S1 = Main student participant
  Each file was coded with the session number and date and securely stored in an encrypted research repository accessible only to the research team.

2.4. Data Analysis
Data were analysed using thematic analysis (Braun & Clarke, 2019) through six iterative stages: familiarisation with data through repeated reading of transcripts and field notes, initial coding to identify significant patterns and meanings, generating preliminary themes according to the mindful–meaningful–joyful framework, reviewing and refining the thematic structure, defining and naming themes operationally, and producing the analytical report, including thematic narratives and interaction matrices between r and s1.

To ensure credibility and trustworthiness, several validation techniques were applied: Source and method triangulation (worksheet, observation, interview), Member checking with participants to verify interpretations, Peer debriefing with two university lecturers in mathematics and educational psychology, and Audit trail documentation to record the entire analytical process.

Additionally, intercoder reliability was established by having two independent researchers re-code the transcripts. The resulting intercoder agreement was 0.91 (very high category), demonstrating stable and reliable interpretation of the qualitative data.

3. Results

3.1 Mapping of Students with MID and Their Numerical Difficulties

The initial psychological assessment using the CFIT identified eight students (13.1% of the total 61 students) categorised as Mild Intellectual Disability (MID) with IQ scores ranging from 50–69. Subsequent diagnostic numeracy tests revealed varying levels of difficulty in interpreting contextual mathematical problems.

Table 1: Distribution of Numerical Difficulties among Students with MID (n = 8)

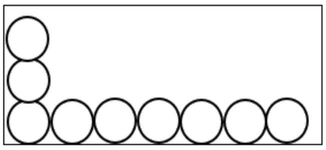
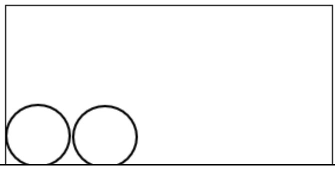
Type of Difficulty	Frequency	Percentage	Description
Misunderstanding the problem (reading/comprehension error)	6	75%	Misinterpreting contextual information, failing to identify given and required data.
Failure to formulate mathematical operations	5	62.5%	Unable to connect problem context to mathematical representation.
Computational or procedural errors	4	50%	Incorrect arithmetic calculation (e.g., 60 × 30 × 45 = 8 100 instead of 81 000).
Spatial reasoning error	3	37.5%	Difficulty visualising object arrangement or shape comparison.
No response	2	25%	Left items blank or responded 'don't know.'

As illustrated in Figure 1, the majority of students experienced difficulties at the problem comprehension stage, indicating that conceptual understanding and contextual reasoning were the most significant barriers.

3.2 Prior Mathematical Knowledge of Students with MID

The following section presents a summary of the students' responses, with a particular focus on those categorized as MID.

Table 2: Distribution of MID students' answers in the initial knowledge test

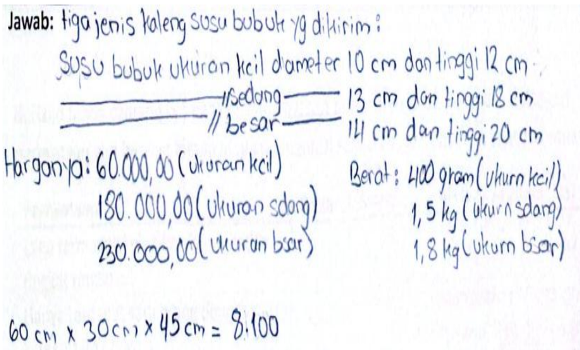
No	Problems	Number and percentage of MID students		
		Answer correctly	**Answered wrong**	**No answer**
1	Draw a picture or sketch of a closed cylinder. Then, mark the diameter and height of the cylinder.	-	8 (100%)	-
2	Is the cylinder shown in the picture in an upright position or not (lying position)? Explain why!	7 (87,5%)	-	1 (12,5%)
3	Have you ever seen a tube-shaped can of milk? Name the brand if so. Where did you see it?	8 (100%)	-	-
4	Determine the maximum number of circles of the same size to fill the box below. 	3 (37,5%)	5 (62,5%)	-
5	Determine the maximum number of circles of the same size to fill the box below. 	4 (50%)	3 (37,5%)	1 (12,5%)
6	If the price of one small can of powdered milk weighing 400 grams is Rp 60 000.00, how much would five small cans of powdered milk cost?	4 (50%)	2 (25%)	2 (25%)
7	If the price of one medium can of powdered milk weighing 1.5 kg is Rp 180 000.00, how much do three small cans of powdered milk cost?	1 (12,5%)	2 (25%)	5 (62,5%)

Based on the findings from students with mild intellectual disabilities (MID), there was notable variation in their understanding of geometric concepts and contextual numeracy. In the first task, all students (100%) were unable to accurately draw a closed cylinder with correct markings for diameter and height, indicating that formal visual representation remains a significant challenge. However, in the second task, the majority of students (87.5%) successfully identified the correct position of the cylinder and provided logical reasoning, suggesting that their spatial awareness and visual orientation are relatively well developed. When asked to name a brand of canned milk shaped like a cylinder, all students (100%) responded correctly, demonstrating a strong connection between mathematical concepts and real-life experiences.

In contrast, students' estimation and visualisation abilities in the context of shape-filling tasks (Items 4 and 5) yielded varied results, with only 37.5% to 50% answering correctly. This indicates that their conceptual understanding of area remains limited. Regarding contextual numeracy—such as calculating prices based on quantity (Items 6 and 7), only half of the students provided correct answers for the simpler task, and just 12.5% succeeded in solving the more complex one. These findings suggest that students' arithmetic skills in real-life contexts are still underdeveloped and require gradual, scaffolded intervention.

3.3 Deep Scaffolding Process and Student Responses

Out of eight students categorised as MID, six provided responses while two did not. Among the six, none of the answers reached the final solution. The most prominent error observed was that MID students were unable to comprehend complex numeracy problems and failed to model them into simpler forms. One MID student who participated in this study stated, 'This question made me completely confused, and I didn't understand it at all.' The student gave the following response:



Anwers: Three types of powdered milk cans were shipped:
Small powdered milk: 10 cm in diameter and 12 cm in height
Medium powdered milk: 13 cm in diameter and 18 cm in height
Large powdered milk: 14 cm in diameter and 20 cm in height

Price: 60 000.00 (small size)
180 000.00 (medium size)
230 000.00 (large size)
Weight: 400 grams (small size)
1.5 kg (medium size)
1.8 kg (large size)

$60 \text{ cm} \times 30\text{cm} \times 45\text{cm} = 8.100$

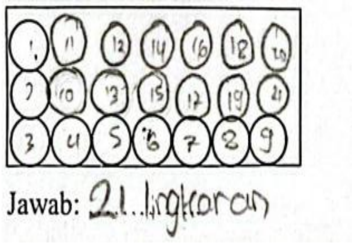
Figure 1: S1's answer before deep scaffolding was carried out

Based on the response, S1 demonstrated an understanding of the problem by rewriting the information provided in the question and attempting to solve it by multiplying 60, 30, and 45 to obtain 8 100 (instead of the correct answer, 81 000). The researcher then applied mindful scaffolding by probing the source of the error and guiding S1 to become aware of the mistake. Through the student's responses and the exploratory dialogue during the interview, S1 became aware of several key aspects: the existence of three types of canned milk with different sizes and prices; the awareness of the dimensions of the cardboard box; the understanding that the task required placing the cans into the box; and the realisation that the cans should be placed in an upright position. S1 also shared a personal experience of seeing canned milk arranged upright during a visit to a supermarket and mentioned a specific milk brand. Furthermore, S1 recognised that more than one can could be placed inside the box. Ultimately, S1 acknowledged that the answer provided was incorrect and expressed uncertainty about how to properly solve the problem.

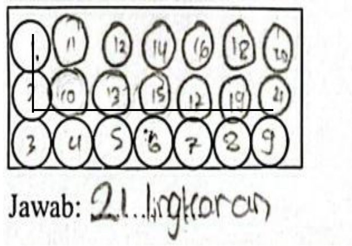
The researcher then implemented meaningful scaffolding by exploring the student’s prior knowledge related to the problem at hand. In this context, it was found that Subject 1 (S1) had concrete experience with cylindrical powdered milk cans placed in an upright position. S1 was also familiar with the elements of a cylinder, such as height and diameter, and demonstrated an understanding of the concept of area in two-dimensional shapes. Furthermore, when calculating how many identical small circles could fit into a rectangle, S1 employed a “counting all” strategy, tallying each circle individually. S1 showed limited fluency in multiplication and struggled with column multiplication. To address this, the researcher provided meaningful scaffolding aimed at guiding S1 toward more efficient calculation strategies. The following is an excerpt from the interview conducted.

R : What is this shape?
S1 : Rectangle.
R : What is the formula for the area of a rectangle?
S1 : Length times width.
R : Which is the length and which is the width?
S1 : This is the length, this is the width, sir. (Pointing to a correctly drawn rectangle)
R : To determine the number of circles in the rectangle, is there another way besides counting all the circles one by one? (MeScaff 1)
S1 : (Pauses.) I don't know, sir. I count them all by numbering them one by one.
R : Look at the length, and how many circles are there in the width?
S1 : There are seven in the length, sir. There are three in the width.
R : So what?
S1 : Multiply them, sir. The result is twenty-one. (Calculating using the 7×1, 7×2, 7×3 method)
R : Which method is more efficient for calculating the number of circles in a rectangle? (Miscaff 1)
S1 : By multiplying, sir.

Based on the interview excerpt above, S1 was able to reflect that the previous method was inefficient, and calculating by direct multiplication was considered more efficient even though S1 had not yet memorised multiplication. The following is S1’s work, which shows a change in the method used to calculate the number of circles within a rectangle.



Gambar 2a: S1 counted the number of circles using the counting all method



Gambar 2b: S1 calculated the number of circles using the concept of the area of a rectangle

Based on this initial data, the researcher carried out the following meaningful scaffolding steps:

R : Try working on this problem again (pointing to the numeracy problem). Read it, understand it and try to do it. (MeScaff 2)
S1 : (reading the problem and trying to understand it). No, sir, I don't know how.

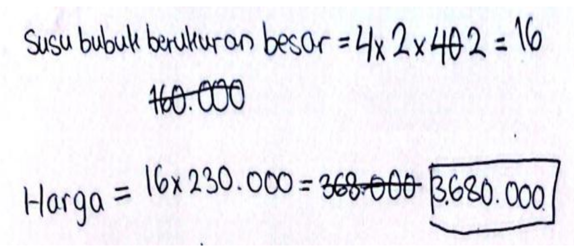
Based on the interview excerpt above, S1 was unable to self-reflect on his prior knowledge to solve the numeracy problems he faced. The researcher then provided further meaningful scaffolding, as follows:

R : What types of canned milk can be put into a carton?
S1 : Hmm... the large ones, maybe.
R : Yes... the large ones. Look at the length of the carton. How many cans of milk can it hold? (MeScaff 3)
S1 : (Thinks for a long time). I don't know, sir.
R : If one can is put into the carton, how many centimetres does it take up?
S1 : (Thinks for a long time and doesn't answer).
R : Look at the closed tube we made earlier. Imagine one tube, like a milk can, being put into the carton. How many centimetres would it take up? (MeScaff 4)
S1 : (Thinks for a long time and doesn't answer).
R : Look at the diameter of the large milk can.
S1 : Fourteen packs. 2
R : So?
S1 : Oh yeah, fourteen packs for one can of milk.
R : How many for two?
S1 : Twenty-four packs. (Tries several times). Twenty-eight packs.
R : If it's up to 60 centimetres, that means how many cans can be filled?
S1 : (counting for quite a long time). Three cans sir.
R : Just three cans?
S1 : (counting). Four pack.
R : Yes, four cans. How many cans aside? (cardboard width)
S1 : Two cans, sir.
R : Yes, two cans. What if it goes up?
S1 : (thinking for a long time). Two cans sir.

Based on the interview excerpt above, two instances of meaningful scaffolding were conducted. In the third instance (MeScaff 3), the researcher guided Subject 1

(S1) to connect prior knowledge about determining the number of circles that can fit into a rectangle with the task of calculating how many milk cans could be placed along the length of a cardboard box. Initially, S1 struggled with the calculation. However, the researcher prompted S1 to focus on the diameter of the large milk can and to visualise placing the cans inside the box. S1 was able to provide the correct answer: 14.

Following this, the researcher encouraged S1 to determine the maximum number of large milk cans that could fit along the length of the box. S1 encountered difficulty when calculating 14 + 14, initially stating the result as 24. Upon re-evaluation, S1 corrected the answer to 28. At first, S1 counted only three cans, but after further prompting, revised the response to four cans and confirmed that no more than four could fit. S1 then easily determined the number of milk cans that could fit along the width of the box and along its height, although there was a moment of hesitation regarding whether to use the diameter or the height of the cylinder. When writing the final answer, S1 initially wrote "40," interpreting it as two cans or "20 + 20", but upon clarification, corrected the answer to "2". The following section presents S1's completed work.

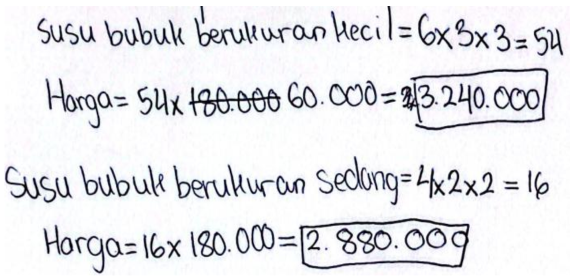


Large powdered milk = 4 × 2 × ~~40~~ 2 = 16
~~160.000~~

Price = 16 × 230.000 = ~~368.000~~ 3.680.000

Figure 3: S1 Answers in Determining the Amount and Price of Large-Sized Powdered Milk

Based on the response above, Subject 1 (S1) was able to calculate the number of large milk cans that could fit into the box, total 16 (although earlier responses included errors such as stating that four times two equals eight, and eight times two equals twelve). S1 then proceeded to calculate the total price by multiplying 16 by 230 000. During this process, S1 encountered difficulties with column multiplication involving a three-digit number and a two-digit number (230 × 16). After arriving at the intermediate result of 3 680, S1 initially wrote 368 000. Upon being prompted to reflect, S1 was able to revise the answer correctly to 3 680 000. Subsequently, S1 independently determined the quantity and price of small and medium-sized powdered milk cans that could be placed inside the box. The results of S1's work are presented below.



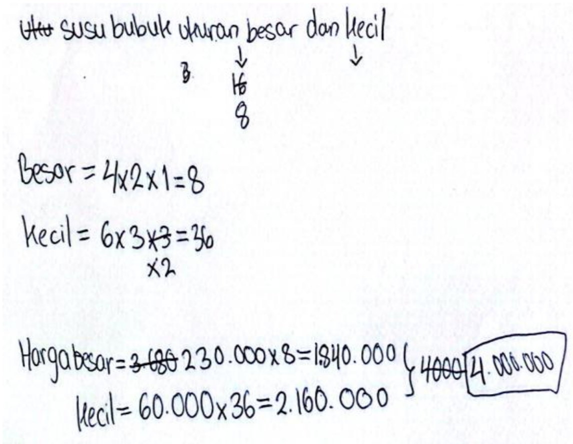
Small powdered milk = 6 × 3 × 3 = 54
Price= 54 × ~~180.000~~ 60.000 = 3.240.000
Medium-sized powdered milk= 4 × 2 × 2 = 16
Price= 16 × 180.000 = 2.880.000

Figure 4: S1's answer in determining the quantity and price of small and medium-sized powdered milk

Based on S1's responses, it was evident that the student first calculated the number of small powdered milk cans as 6 × 3 × 3 = 54, and then determined the total price as 54 × 60 000 = 3 240 000. For the medium-sized powdered milk cans, S1 calculated the quantity as 4 × 2 × 2 = 16, with a total price of 16 × 180 000 = 2 880 000. Although the calculations were correct, S1 encountered recurring difficulties during the process, particularly with column multiplication and fluency in basic multiplication, which resulted in a prolonged problem-solving time. The meaningful scaffolding provided aimed to give S1 the opportunity to connect prior knowledge to the problem at hand, even though the process required considerable time and support.

R : Have all the calculations completed the problem?
S1 : Yes, it's complete. The answer is 3 680 000.
R : Are you sure?
S1 : Yes, sir.
R : Please review the problem again. Are there any other possibilities?
S1 : (Thinks for a long time.) I don't think so, sir.

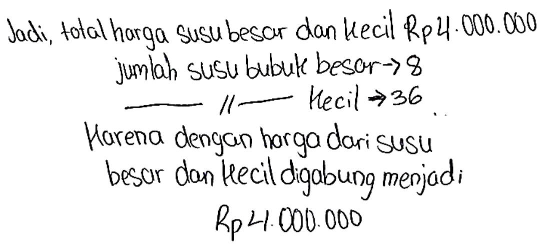
Based on the interview excerpt above, Subject 1 (S1) initially had no idea how to combine milk cans of different sizes to be placed inside the box. The researcher then applied mindful scaffolding by encouraging S1 to think more critically about the problem and visualise the scenario: if the large milk cans were placed at the bottom, what other sizes of milk cans could potentially be added afterward? Through this guided questioning, S1 was able to reflect and conclude that a possible solution would involve a combination of large and small powdered milk cans. The following section presents S1's completed work.



Large and small sized powdered milk
Large= 4 × 2 × 1 = 8
Small= 6 × 3 ~~× 3~~ = 16 × 2
Price of the Large= ~~3.680~~ 230.000 × 8 = 1.840.000
Price of the Small= 60.000 × 36 = 2.160.000
Total ~~4 000~~ 4.000.000

Figure 5: S1's answer in determining the amount and quantity of large and small sized powdered milk

Based on the response above, Subject 1 (S1) calculated the number of large powdered milk cans as 4 × 2 × 1 = 8, and the number of small powdered milk cans as 6 × 3 × 2 = 36. Although S1 initially made an error by assuming three layers of small cans could be stacked vertically—overlooking the presence of large cans beneath—this was corrected through mindful scaffolding, prompted by the question, "Is it really three?" S1 then calculated the total price of the large powdered milk cans as 1 840 000 and the small ones as 2 160 000, resulting in a combined total of 4 000 000. After exploring four possible solutions, S1 ultimately determined the final answer as follows:



So, the total price of the large and small milk bottles is Rp 4 000 000.
Number of large powdered milk bottles → 8
Number of small powdered milk bottles → 36
Because the price of the large and small milk bottles combined is Rp 4 000 000.

Figure 6: S1's answer in determining the conclusion for the final answer

S1's answer indicated that he was able to determine that the total price of large and small milk powders was the highest, with eight large milk powders and thirty-six small milk powders. After finding the answer, the researcher checked S1's mood during the deep scaffolding stage.

R : From one to five, how do you feel? If one describes your mood as very unpleasant and five as very pleasant? And tell us why you chose that number?

S1 : Six, sir. Because I understand, from not understanding before to understanding, so now I'm relieved.

S1 described his mood as exceeding the maximum of six, where he felt very happy and relieved, he was able to get out of the problem he was facing and was able to find the correct answer. The process that S1 went through was quite difficult because he had insufficient initial knowledge, such as not memorizing multiplication, not understanding the concept of area, and not fluently doing stacked multiplication. S1 also stated that he had never encountered a numeracy problem like the one he was facing, but was finally able to solve it with his great effort.

The following is an overview of the deep scaffolding outcomes, consisting of: Type of Error Before Intervention, Intervention Strategies and Observable Improvement.

Table 3: Type of Error Before Intervention, Intervention Strategies dan Observable Improvement

Scaffolding Stage	Type of Error Before Intervention	Intervention Strategies	Observable Improvement
Mindful	Misinterpretation of numerical data; conceptual confusion	Guided questioning; reflection on problem meaning	Awareness of mistake; correction of calculation (from 8 100 → 81 000)
Meaningful	Weak spatial reasoning; failure to visualise arrangement	Linking real-life context (milk cans in boxes)	Improved spatial visualisation and logical reasoning
Joyful	Low motivation and confidence	Reflective dialogue; positive reinforcement	Increased engagement, confidence and verbal explanation of reasoning

The Deep Scaffolding intervention consisted of three interconnected stages, Mindful, Meaningful and Joyful Scaffolding, implemented over six individual sessions with one participant (S1). Mindful Scaffolding focused on identifying and reflecting on cognitive errors. S1 initially misread numerical information (multiplying $60 \times 30 \times 45 = 8\,100$) and was unaware of the conceptual gap. Through guided questioning ("Why did you use that method?" "What does the problem ask you to find?"), S1 became aware of the error source and corrected the reasoning.

Meaningful Scaffolding aimed to link prior experience to mathematical meaning. S1 recalled observing upright milk cans in a supermarket and realised that cans could be arranged vertically in a box—connecting real-world schema with spatial reasoning. Joyful Scaffolding emphasised reflection and emotional engagement. By the final session, S1 expressed confidence and enjoyment ("I understand better now; it's fun to find the right answer.").

4. Discussion

The findings of this study provide a comprehensive understanding of how the implementation of deep scaffolding based on Realistic Mathematics Education (RME) facilitates conceptual and emotional transformation among students with mild intellectual disabilities (MID) in solving contextual numeracy problems. The observed results reveal that scaffolding not only supports the cognitive process but also nurtures motivational and emotional engagement that sustains learning persistence. These findings are consistent with prior studies demonstrating that scaffolding significantly enhances conceptual understanding and mathematical reasoning (Reinhold et al., 2020; Esparcia, Piñero, & Futalan, 2024) but extend the discussion by emphasising the affective dimension as a determinant of cognitive growth.

In contrast to conventional scaffolding approaches that emphasise procedural guidance or digital assistance (Malik, Abdi, Wang, & Demszky, 2025; Long, Bouck, & Domka, 2020), deep scaffolding in this study demonstrated how mindful, meaningful and joyful phases can trigger metacognitive awareness and emotional readiness for learning. Subject 1 (S1) was able to progress from misunderstanding the problem to constructing logical strategies and finally solving complex tasks, a transformation reflecting an internalisation of mathematical reasoning grounded in empathy and reflection.

This finding indicates that cognitive transformation among MID students is highly dependent on emotional regulation and motivational support. Zehner et al. (2024) argue that emotional competence predicts early numeracy success, supporting the view that learning is both an intellectual and affective process. The integration of emotional attunement in scaffolding thus acts as a bridge between cognitive limitations and conceptual mastery. The present findings also reaffirm Freudenthal's theory that mathematical understanding evolves through progressive mathematisation—from situational reasoning to formal abstraction (Van den Heuvel-Panhuizen & Drijvers, 2020).

However, for students with MID who experience limitations in working memory and abstraction (Brankaer, Ghesquière, & de Smedt, 2013), this process must be enriched by systematic, emotionally responsive scaffolding. Deep scaffolding operationalises this progression by embedding teacher–student dialogue that links real-world contexts, reflection and confidence-building. Such findings are in line with studies by Root et al. (2018) and Hord (2022), which highlight that contextualised and affective learning environments enable students with intellectual disabilities to transfer knowledge effectively from familiar situations to abstract mathematical representations.

Interestingly, the current results slightly diverge from Cheong, Walker, and Rosenblatt (2017), who reported limited generalisation ability among learners with MID. In contrast, when emotional reflection and mindful questioning were applied, S1 was able to generalise reasoning patterns and recognise connections between previously unrelated tasks. This suggests that emotional scaffolding, through empathetic dialogue and reflective questioning, may be the critical factor missing in traditional scaffolding approaches. Moreover, the student's enthusiasm and sense of relief upon achieving understanding illustrate the importance of joyful learning, as emphasised by Clements and Sarama (2018), who describe that affective engagement strengthens conceptual retention in mathematical learning trajectories.

Despite these promising results, several limitations must be acknowledged. The study involved only one focal participant (S1), which restricts the generalizability of findings. The relatively short intervention period (three weeks) also limits the ability to capture long-term cognitive and affective changes. Furthermore, as a qualitative single-case design, the interpretations are contextually bound and may not represent all MID learners. Nevertheless, the methodological rigour, including triangulation, member checking, and intercoder reliability (0.91), ensures the credibility and dependability of the results. These limitations open opportunities for future research to employ mixed-method or longitudinal approaches to examine the sustainability and transferability of deep scaffolding effects.

The findings carry both theoretical and practical implications. Practically, teachers are encouraged to design learning experiences that incorporate mindful scaffolding to help students recognise cognitive errors, meaningful scaffolding to connect mathematical concepts with familiar daily-life contexts, and joyful scaffolding to sustain motivation and confidence. This RME-based scaffolding model can help teachers transform numeracy learning from procedural memorisation into reflective, emotionally supportive dialogue.

However, implementing such practices may face challenges, including time constraints, insufficient teacher training and the need for emotional sensitivity when working with MID students. Theoretically, this study contributes to the expansion of inclusive pedagogy by demonstrating that scaffolding is not solely a cognitive tool but also an affective–motivational process that fosters autonomy, reflection and self-efficacy among learners with special needs. The mindful–meaningful–joyful framework proposed here extends the theoretical understanding of how RME principles can be operationalised through empathetic and culturally contextualised interactions.

For future research, scholars are encouraged to examine deep scaffolding through broader samples and quantitative measures to evaluate its statistical effectiveness. Longitudinal studies may reveal how sustained emotional engagement affects the retention and transfer of numeracy skills. Additionally, the development of diagnostic instruments to assess both cognitive and affective dimensions of scaffolding would strengthen empirical evidence. Research in other learning domains such as science and literacy could further test the adaptability of this model across disciplines.

From a methodological standpoint, reflection on the challenges during implementation, such as student anxiety, slow response, and difficulties in abstract reasoning, underscores the importance of trustful teacher–student relationships. These reflections confirm that deep scaffolding promotes dependable and authentic learning experiences, even within the constraints of inclusive classrooms. Ultimately, this study underscores that RME-based deep scaffolding is not merely an instructional support but a transformative pedagogical philosophy that empowers students with mild intellectual disabilities as capable and dignified learners who can engage meaningfully with mathematics and the world around them.

5. Conclusion

This study concludes that deep scaffolding based on Realistic Mathematics Education (RME) effectively remediates errors made by students with mild intellectual disabilities (MID) in solving numeracy problems. Through three interrelated phases—mindful, meaningful and joyful—students showed significant progress in identifying and correcting reasoning errors, understanding contextual problems and developing more logical and reflective problem-solving strategies.

The process not only improved conceptual understanding but also strengthened metacognitive awareness and emotional engagement, indicating that empathetic and contextualised instructional support enables MID students to build more meaningful and sustainable numeracy competence. Theoretically, this study contributes to inclusive education by positioning deep scaffolding as a multidimensional model that bridges RME principles with the realities of special education and extends Freudenthal's concept of progressive mathematisation by emphasising the importance of metacognitive and emotional scaffolding.

Practically, this research provides clear implications for teachers, policymakers and researchers. Teachers are encouraged to apply mindful, meaningful and joyful scaffolding to address students' cognitive errors, connect mathematical ideas with real-life contexts, and sustain learning motivation. Policymakers and teacher education programmes can use this framework to promote empathetic, adaptive and reflective numeracy instruction within inclusive settings. Although this study offers valuable insights, it acknowledges certain limitations, including its single-case qualitative design, short intervention duration and limited generalizability.

Future research should employ mixed-method or experimental approaches with larger samples, develop assessment tools that capture both cognitive and emotional progress, and explore the application of RME-based deep scaffolding in other subjects such as science and literacy. In essence, deep scaffolding is not merely an instructional strategy but a transformative philosophy that unites cognitive challenge, emotional support and contextual relevance—redefining inclusive education as meaningful participation and empowerment for all learners.

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